What do we mean by correlation?
It shows the degree of linear association between two variables
Nifty ↑ Sensex ↑
Nifty ↓ Sensex ↓

Why do we study?
"Stock Market involve wide application of Correlation"

CH-17

CORRELATION & REGRESSION

[Avg. w. 5 marks]

Correlation

Introduction

Correlation Coefficient (r)

x	y	x	y	x	y
↑	↑	↑	↓	↑	✗
↓	↓	↓	↑	↑	✗

positive correlation $0 < r \leq 1$

Negative Correlation $-1 \leq r < 0$

No Correlation $r = 0$

$$\boxed{-1 \leq r \leq 1}$$

$r = -1$ Perfectly negative Correlation

$-1 < r < 0$ Negative Correlation

$r = 0$ No Correlation

$0 < r < 1$ positive Correlation

$r = +1$ Perfectly Positive Correlation

Straigh Line Proportional always

TYPE I: $ax + by + c = 0$

$r_{xy} = -1$ or $+1$

$2x + y = 5$

$+2x = -y + 5$ Ans -1

$-2x + 3y = 10$

$-2x = -3y + 10$ Ans $+1$

If sign of two variables are same then ans. is $+1$ or opposite then ans. is -1

What do we mean by correlation?
It shows the degree of linear association between two variables
Nifty ↑ Sensex ↑
Nifty ↓ Sensex ↓

Why do we study?
"Stock Market involve wide application of Correlation"

CH-17
CORRELATION & REGRESSION

[Avg. w. 5 marks

Correlation depends upon

- Direction
  - Same: +ve, $x\uparrow\downarrow$ $y\uparrow\downarrow$
  - Opposite: −ve, $x\uparrow\downarrow$ $y\downarrow\uparrow$
- Proportion
  - Yes: 1, Perfect Correlation
  - No: Not 1, No Correlation

x
α
α
Correlation
$r=0$

$r=-1$ Perfectly negative Correlation
$-1<r<0$ Negative Correlation
$r=0$ No Correlation
$0<r<1$ positive Correlation
$r=+1$ Perfectly Positive Correlation

Straigh Line Proportional always

TYPE 1: $ax+by+c=0$
$r_{xy}=-1$ or $+1$

$2x+y=5$
$+2x=-y+5$ $\boxed{\text{Ans } -1}$

$-2x+3y=10$
$-2x=-3y+10$ $\boxed{\text{Ans } +1}$

If sign of two variables are same then ans. is +1 or opposite then ans. is −1

TYPE 2:

x	y	z
↑	↑	↓
↓	↓	↑

$r_{xy} = 0.59$

$2y + 5z = 10$

$r_{xz} = -0.59$ Ans

If the signs are same then ans is positive & if opposite then ans is negative

$r_{ab} = -0.26$

$3a - 2c = 10$

$r_{bc} = -0.26$ Ans

Bivariate Data → Analysis of two datas at one time or table.

p - column

q - rows

Maths → / Stats ↓	0-10	10-20	20-30	30-40
0-10	10	12	15	16
10-20	8	10	9	10
20-30	8	11	6	3

- No. of cells $= p \times q$
  $= 4 \times 3 = 12$
- No. of Marginal Distributions $= 2$
- No. of Conditional Distributions $= p + q$
  $= 4 + 3 = 7$

MD 1 (Maths)

0-10 → 26
10-20 → 33
20-30 → 30
30-40 → 29
118

MD 2 (Stats)

0-10 → 53
10-20 → 37
20-30 → 28
118

Maths 10-20

Stats

0-10	12
10-20	10
20-30	11

Methods of Correlation Analysis

- Scattered Diagram
- Karl Pearson's Coefficient of Correlation
- Spearman's Rank Correlation Coefficient
- Coefficient of Concurrent Deviation

Scattered Diagram → TREND

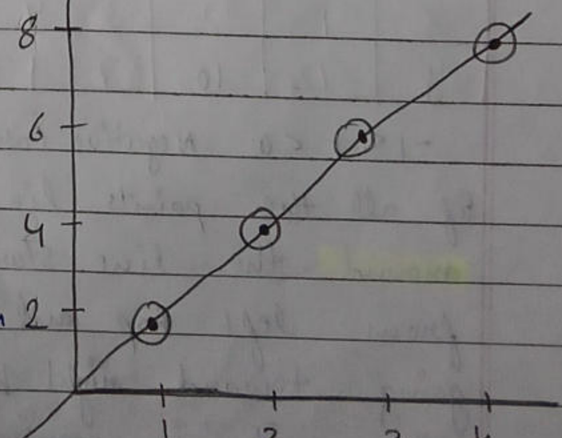
Karl Pearson's Coefficient of Correlation, Spearman's Rank Correlation Coefficient, Coefficient of Concurrent Deviation → NUMERICAL VALUE

1. Scattered Diagram → If all the points on scattered diagram lie on the line starting from left bottom and going toward top right then

x	1	2	3	4
y	2	4	6	8

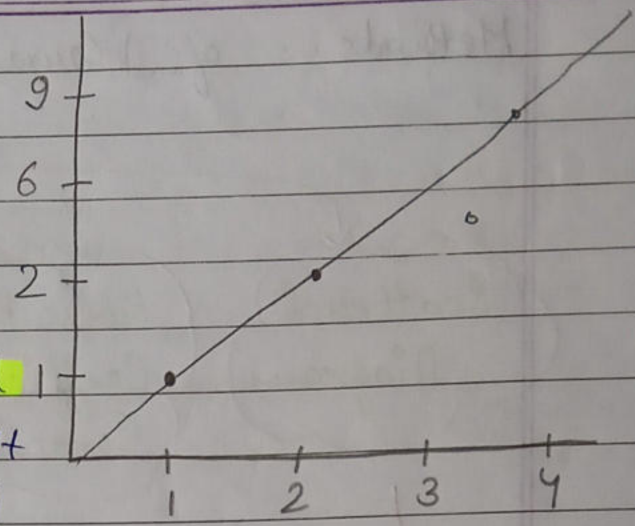
$r = +1$

Perfectly Positive Correlation



→ This method only tells us the nature of correlation & not the magnitude

x	1	2	3	4
y	1	2	6	9

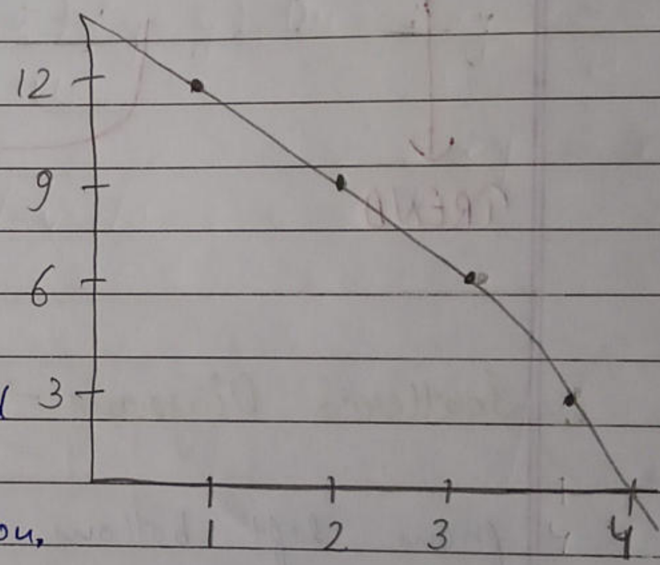


$0 < r \leq 1$

Positive Correlation

If all the points on scattered diagram lie around the line starting from left bottom and going toward top right

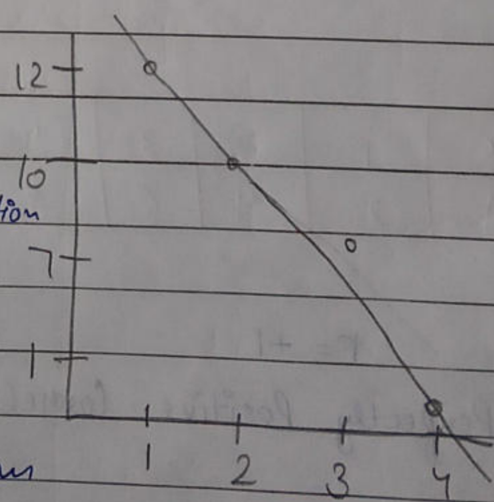
x	1	2	3	4
y	12	9	6	3



$r = -1$

If all the points lie on the line starting from left top and going toward right bottom then it is perfectly negative correlation.

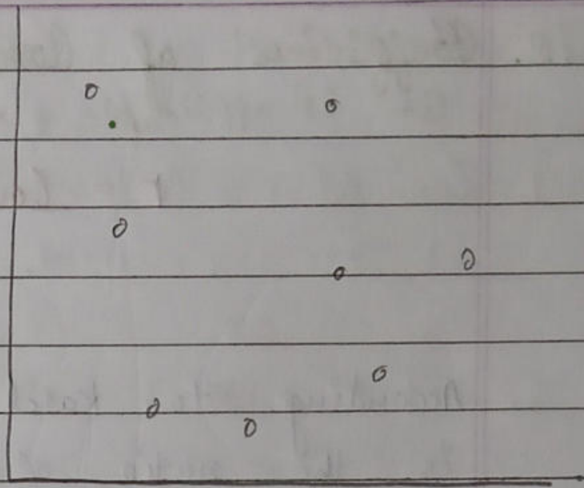
x	1	2	3	4
y	12	10	7	1



$-1 \leq r < 0$ Negative Correlation

If all the points lie around the line starting from left top and going toward right bottom then it is ~~perfectly~~ Negative correlation.

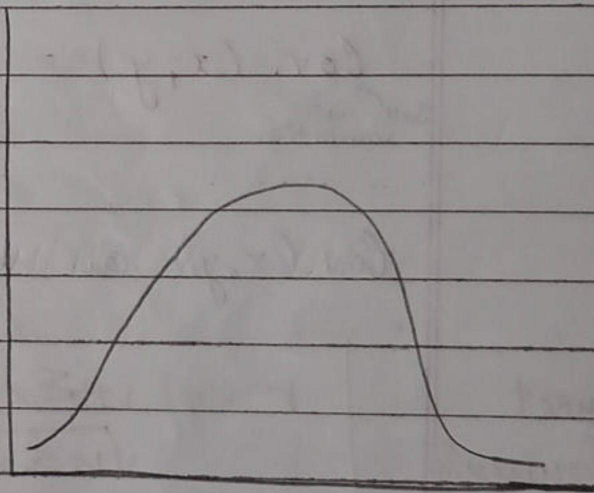
x	1	2	3	4
y	10	6	19	1



$r = 0$ [Spurious Corr.]

No Correlation

x	1	2	3	4	5
y	2	4	10	4	2



Curvi Linear Correaltion

$r = 0$

KP

2. Coefficient of Correlation (Product Moment Corr. Coeff.)

It is applicable only for Linear variable

$$r = \frac{\text{Covariance of } xy}{\sigma_x \times \sigma_y} = \frac{Cov(x, y)}{\sigma_x \times \sigma_y}$$

According to Karl Pearson, coefficient of correlation is the ratio of covariance of xy to the product of standard Deviations of x & y.

$$Cov.(x, y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} \text{ Or } = \frac{\Sigma xy}{n} - \frac{\Sigma x}{n} \cdot \frac{\Sigma y}{n}$$

Sath me variation

$$= \frac{\Sigma xy}{n} - \bar{x} \cdot \bar{y}$$

Cov (x,y) can be +ve/-ve/0

Direct formula of KP

$$\boxed{r = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n\Sigma y^2 - (\Sigma y)^2}}}$$

E.g. 1

x	y	xy
2	10	20
5	12	60
8	14	112
10	18	180
25	54	372

$\bar{x} = 6.25$

$\bar{y} = 13.5$

$n = 4$

$$Cov.(xy) = \frac{\Sigma xy}{n} - \bar{x} \cdot \bar{y}$$

$$= \frac{372}{4} - 6.25 \times 13.5$$

$$= 8.625$$

E.g. 2

x	y	xy
1	5	5
3	8	24
4	10	40
6	12	72
14	35	141

$\bar{x} = 3.5$

$\bar{y} = 8.75$

$n = 4$

$$Cov.(xy) = \frac{\Sigma xy}{n} - \bar{x} \cdot \bar{y}$$

$$= \frac{141}{4} - 3.5 \times 8.75$$

$$= 4.625$$

Eg. 3-

x	y	x^2	y^2	xy
1	5	1	25	5
3	8	9	64	24
4	10	16	100	40
6	12	36	144	72
14	35	62	333	141

$\bar{x} = 3.5$

$\bar{y} = 8.75$

$n = 4$

$$r = \frac{n\Sigma xy - \Sigma x . \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2} . \sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{4 \times 141 - 14 \times 35}{\sqrt{4 \times 62 - 196} . \sqrt{4 \times 333 - 1225}}$$

$$= \frac{74}{\sqrt{52 \times 107}}$$

$\longrightarrow$ D.O.C. = $52 \times 107 = \sqrt{\ }$ $\div = \times 74$

$= 0.9920$

Kuch angrezi Baatein

$\Sigma (x-\bar{x})(y-\bar{y})$ = Sum of product of deviations

$\Sigma (x-\bar{x})^2$ = Sum of squares of deviation

Σx^2 = Sum of squares of x

$(\Sigma x)^2$ = Square of sum of x

Properties of Correlation Coefficient (K.P.)

1. Correlation Coefficient is a unit free quantity i.e. It has no unit.

2. $-1 \leq r \leq 1 \Rightarrow r^2 \leq 1$ $[\sigma_x \sigma_y \geq Cov.(x,y)]$

3. K.P. Coefficient is independent of both change in origin as well as change in scale but the sign needs to be considered.

$r_{xy} = 0.58$

$+2x \quad +y+5 \quad r = 0.58$

Same sign → No change

Opp. sign → Change the sign

o It is the most accurat & acceptable method of correlation

$\frac{\text{Explained Var.}}{\text{Total Var.}} \times 100$

4. Coefficient of determination $= r^2$ (Explained variance)

5. Coefficient of Non-determination $= 1 - r^2$ (Unexplained variance)

$\frac{\text{Unexplained Var.}}{\text{Total Var.}} \times 100$

6. Nature of correlation entirely depends upon nature of covariance.

If cov. = -ve, r = -ve

If cov. = +ve, r = +ve

3. Spearman's Rank Coefficient of Correlation

It is represented by r_0

(i) It is important method for ordinal data / qualitative data

High to Low or Low to High } Same Rank

$$r_0 = 1 - \frac{6 \Sigma d^2}{n(n^2-1) \text{ or } (n^3 - n)}$$

(ii) If the Rank are exactly opposite then $r_0 = -1$

Σd^2 = Sum of Squares of difference in ranks of x & y.

(iii) If the ranks are exactly same then r_0 will always be 1

it will always 0

E.g.

x	y	R_x	R_y	$d = R_x - R_y$	d^2
15	5	3	5	−2	4
18	13	2	1	+1	1
10	10	4	2	+2	4
9	6	5	4	+1	1
20	9	1	3	−2	4
				$\Sigma d = 0$	$\Sigma d^2 = 14$

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 14}{5 \times 24}$$ → D.O.C. = 6×14 ÷ 5 ÷ 24 +/− +1 =

$= 0.3$

Formula for Common Ranks

$$r_0 = 1 - \frac{6\left[\Sigma d^2 + \frac{\Sigma (m^3 - m)}{12}\right]}{n(n^2-1) \text{ or } n^3 - n}$$

m → the times no. repeated or common ranks

Jitno kai beech tie hai

Eg.

x	y	Rx	Ry	d	d^2
40	20	1.5	2	-0.5	0.25
38	20	4.5	2	2.5	6.25
40	18	1.5	4	-2.5	6.25
32	20	6	2	4	16
38	16	4.5	5	-0.5	0.25
39	5	3	6	-3	9
				0	38

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{\Sigma(m^3-m)}{12}\right]}{n(n^2-1)}$$

$$r = 1 - \frac{6\left[38 + \frac{(2^3-2)+(2^3-2)+(3^3-3)}{12}\right]}{6 \times 35}$$

$r = -0.1714$

D.O.C. → $6 + 6 + 24 \div 12 + 38$
$\times 6 \div 6 \div 35$ +/− +1

Example 4 The sum of sq. of difference in ranks of the marks obtained by the 15 students in two different subjects was found to be 80, find the Spearman's Rank Correlation Coefficient.

Solve- $n = 15$

$\Sigma d^2 = 80$

$6 \times 80 \div 15 \div 224$ +/− $+1 = 0.657$

$r = 0.657$

Example 2- If the sum of squares of difference in Ranks of certain number of states was found to be 63 and the Rank Correlation was −0.125.
(a) 9 ~~(b) 7~~ (c) 8 (d) 6

Solve- $\Sigma d^2 = 63$

$r = -0.125$

$$-0.125 = 1 - \frac{6 \times 63}{n(n^2-1)}$$

[Do direct by knock out method]

Example 3- The value of Rank Correlation b/w age & spectacles numbers of 12 persons was found to be 0.55. For one of the candidates the difference in the rank was wrongly taken as 9 instead of 6. If the mistake is rectified find the correct correlation coefficient.

Solve- $n = 12$

$r = 0.55$

$$0.55 = 1 - \frac{6 \Sigma d^2}{12 \times 143}$$

$$\frac{6 \Sigma d^2}{12 \times 143} = 1 - 0.55$$

D.O.C → $1 - 0.55 \times 12 \times 143 \div 6$ $\Sigma d^2 = 128.7$

$128.7 - 9^2 + 6^2 = 83.7$

$6 \times 83.7 \div 12 \div 143$ +/− $+ 1 = 0.707$

4. Cofficcient of Concurrent Deviations

$$r = +/- \sqrt{\frac{2c-m}{m}}$$

applicable if $\sqrt{}$ value is -ve

c → No. of concurrents [count + sign in last column]
m = n-1 , no. of observations compared

Eg: Calculate the coefficient of concurrent Deviations

x	y	Cx	Cy	C = Cx × Cy
58	100	×	×	
64	105	+	+	+
49	98	−	−	+
36	96	−	−	+
39	107	+	+	+
40	109	+	+	+
44	100	+	−	−
59	97	+	−	−

C = 5
m = 8 − 1
= 7

$$r = \sqrt{\frac{2 \times 5 - 7}{7}}$$

$$r = \sqrt{\frac{3}{7}}$$

$$\boxed{r = 0.654}$$

E.g. If the coefficient of concurrent deviation was found to be $1/\sqrt{5}$ and the number of concurrent was 6, then find the number of observations

→ $r = \frac{1}{\sqrt{5}}$

$c = 6$

$n = ?$

$$\frac{1}{\sqrt{5}} = \sqrt{\frac{2 \times 6 - m}{m}}$$

$$\frac{1}{5} = \frac{12 - m}{m}$$

$$m = 60 - 5m$$

$$6m = 60$$

$$\boxed{m = 10}$$

$n = m + 1$

$n = 10 + 1$

$\boxed{n = 11}$ Ans

Regression

We use slope point form to make regression equations

Regression Analysis (Regression Equations)

$x \to$ Given, $y \to$ find — Regression of y on x

$y \to$ Given, $x \to$ find — Regression of x on y

x Advertisement	y Sales
10	130
12	140
15	125
18	135
22	150

? —— 200

25 —— ?

What is diff. b/w corr. & regg.

→ Correlation only tells us that if x increases, whether y will increase or decrease.

However, regression is a step forward it tells the exact value of y, if exact value of x is given.

Regression Line of y on x

$$y = a + b_{yx} X$$

$b_{yx} \to$ Regression Coefficient of y on x

$$b_{yx} = \frac{Cov.(x,y)}{Var.(x)} = \frac{Cov.(x,y)}{\sigma_x^2}$$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x}$$

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

To find a in this equation
Replace $(x, y) \rightarrow (\bar{x}, \bar{y})$

Q.

x	y
10	5
12	8
15	10
18	13
20	15
↓	
25	?

$\Sigma x = 75$
$\Sigma y = 51$
$\Sigma xy = 830$
$\Sigma x^2 = 1,193$
$n = 5$
$\bar{x} = 15$
$\bar{y} = 10.2$

$$y = a + byx\, x$$

$$byx = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$byx = \frac{5 \times 830 - 75 \times 51}{5 \times 1,193 - (75)^2} = \frac{325}{340} = 0.9558$$

$$y = a + 0.9558x$$
$$10.2 = a + 0.9558 \times 15$$
$$\boxed{a = -4.137}$$

D.O.C → 0.9558 × 15 +/− + 10.2

$$y = -4.137 + 0.9558 \times x$$
$$y = -4.137 + 0.9558 \times 25$$
$$\boxed{y = 19.75}$$

Regression Line of x on y

$$x = a + b_{xy}\, y$$

$b_{xy} \rightarrow$ Regression coefficient of x on y

$$b_{xy} = \frac{Cov.(x,y)}{Cov.\, y} = \frac{Cov.(x,y)}{\sigma_y^2}$$

$$b_{xy} = \frac{r\sigma_x}{\sigma_y}$$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

To find a in this equation
Replace $(x,y) \rightarrow (\bar{x}, \bar{y})$

Q.

x	y
5	10
8	15
12	20
13	22
17	28
	↓
	35

$\Sigma x = 55$
$\Sigma y = 95$
$\Sigma xy = 1,172$
$\Sigma y^2 = 1,993$
$\bar{x} = 11$
$\bar{y} = 19$
$n = 5$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{5 \times 1,172 - 55 \times 95}{5 \times 1,993 - (95)^2} = \frac{635}{940} = 0.6755$$

$x = a + b_{xy}\, y$

$11 = a + 0.6755 \times 19$

$\boxed{a = -1.8345}$

$x = a + b_{xy}\, y$

$x = -1.8345 + 0.6755 \times 35$

$\boxed{x = 21.808}$

Q.

x	y
15	6
18	7
20	10
25	9

$y = ?$ $x = 30$

$\Sigma x = 78$ $\quad \bar{x} = 19.5$

$\Sigma y = 32$ $\quad \bar{y} = 8$

$\Sigma xy = 641$ $\quad n = 4$

$\Sigma x^2 = 1{,}574$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{4 \times 641 - 78 \times 32}{4 \times 1{,}574 - (78)^2} = \frac{68}{212} = 0.3207$$

$y = a + b_{yx}\, x$

$a = 1.74635$

$y = 1.74635 + 0.3207 \times 30$

$\boxed{y = 11.36735}$

Q. Ram

	Maths (x)	Stats (y)
Avg	35	41
SD	1.5	2.4

$r = 0.78$ $M = 40$ $S = ?$

$Y = a + b_{yx} X$

$b_{yx} = \frac{r \sigma_y}{\sigma_x}$

$= \frac{0.78 \times 2.4}{1.5} = 1.248$

$a = -2.68$

$Y = a + b_{yx} X$

$Y = -2.68 + 1.248 \times 40$

$\boxed{Y = 47.24}$

Properties of Regression Coefficient

1. b_{yx}, b_{xy} and r all will always have same sign.

2. $b_{yx} = \frac{r\sigma_y}{\sigma_x}$, $b_{xy} = \frac{r\sigma_x}{\sigma_y}$

$$b_{yx} \times b_{xy} = r\frac{\cancel{\sigma_y}}{\cancel{\sigma_x}} \times \frac{\cancel{\sigma_x}}{\cancel{\sigma_y}}$$

$$b_{yx} \times b_{xy} = r^2$$

$$\sqrt{b_{yx} \times b_{xy}} = r$$

Correlation Coefficient is the GM of the two regression coefficient.

Example $b_{yx} = 0.98$, $b_{xy} = 0.75$

$$r^2 = b_{yx} \times b_{xy}$$

$$r = \sqrt{0.98 \times 0.75}$$

$$\boxed{r = 0.85}$$

Example $b_{yx} = -1.2$, $b_{xy} = -0.65$

$$r^2 = b_{yx} \times b_{xy}$$

$$r = \sqrt{-0.65 \times -1.2}$$

$$\boxed{r = -0.88}$$

Example- $\frac{b_{xy} + b_{yx}}{2} \geq r$

$$\frac{b_{xy} + b_{yx}}{2} \geq \sqrt{b_{xy} \times b_{yx}}$$

$$AM \geq GM$$

Definitely True

3. $r^2 \leq 1 = b_{xy} \times b_{yx} \leq 1$

[This condition is used to identify which line is x on y and which line is y on x]

4. The solution of the two regression lines is A.M. of x and y i.e. $(\bar{x}, \bar{y})$

Example- If the two regression lines are these find airthmetic mean of x and y. Find $\bar{x}, \bar{y}$.

$$2x - y = 5$$
$$3x - 2y = 7$$

(A) 1, 2 (B) 2, 3 (C) 3, 1 ✓ (D) 4, 2

[Do by knock out or solve the equation]

5. Regression Coefficients are independent of Δ in origin but they do depend on Δ in scale, Δ in sign.

u & x
$u = \underbrace{a}_{\text{origin}} + \underbrace{b}_{\text{scale}}x$

v & y
$v = \underbrace{c}_{\text{origin}} + \underbrace{d}_{\text{scale}}y$

$b_{yx} \to$ Given $b_{vu} \to ?$ $b_{vu} = \frac{d}{b} \times b_{yx}$

$b_{xy} \to$ Given $b_{uv} \to ?$ $b_{uv} = \frac{b}{d} \times b_{xy}$

Q.1 If $b_{yx} = 1.27$

$2u + 3x = 10$, $3v + y = 7$

find $b_{vu} = ?$

$2u = -3x + 10$, $3v = -y + 7$

$u = \frac{-3x}{2} + \frac{10}{2}$, $v = \frac{-1y}{3} + \frac{7}{3}$

$b = \frac{-3}{2}$, $d = \frac{-1}{3}$

$b_{vu} = \frac{d}{b} \times b_{yx}$

$b_{vu} = \frac{-1/3}{-3/2} \times 1.27$

$b_{vu} = \frac{-1}{3} \times \frac{-2}{3} \times 1.27$

$\boxed{b_{vu} = 0.2822}$

Q.2- If $b_{xy} = 1.5$

$3u + x = 10$, $-3v + 4y = 6$

find $b_{uv} = ?$

$u = \frac{-1x}{3} + \frac{10}{3}$, $v = \frac{4y}{3} - \frac{6}{3}$

$b = \frac{-1}{3}$, $d = \frac{4}{3}$

$b_{uv} = \frac{b}{d} \times b_{xy}$

$b_{uv} = \frac{-1/3}{4/3} \times 1.5 = \frac{-1}{3} \times \frac{3}{4} \times 1.5$

$\boxed{b_{uv} = -0.375}$

6. r^2 is called as coefficient of determination (accounted)

$$r^2 = \frac{\text{Explained Variance}}{\text{Total Variance}}$$

$1 - r^2$ is called as coefficient of non determination (unaccounted)

$$1 - r^2 = \frac{\text{Unexplained Variance}}{\text{Total Variance}}$$

Agriculture production & Rainfall $\rightarrow +0.8$

- $r = 0.8$ $\quad r^2 = (0.8)^2 = 0.64 = 64\%$
- $r = 0.55$ $\quad$ COD $= 30.25\%$
  COND $= 69.75\%$

Q. If $r = 0.62$ then how much percentage of variation is accounted for?

C.O.D. $= (0.62)^2 \times 100 = 38.44\%$

COND $= (1 - r^2) \times 100 =$
$= (1 - 0.62^2) \times 100$
$= 61.56\%$

7. If $r = -1$ or $+1$, then the two regression lines are coincident (ek ke upar ek)
If $r = 0$, then the two regression lines are perpendicular to each other.

8. The slope of the line y on x is 'b_{yx}'
The slope of the line x on y is '$\frac{1}{b_{xy}}$'

Example $5x + 3y = 10$ then y on x is

$3Y = -5x + 10$ $\quad$ $Y = a + b_{yx}\, x$

$Y = \frac{-5x}{3} + \frac{10}{3}$ $\quad$ $b_{yx} = \frac{-5}{3}$

Example $x - 3y = 12$ then x on y is

$x = 3y + 12$ $\quad$ $X = a + b_{xy}\, y$

$b_{xy} = 3$ $\quad$ Slope $= \frac{1}{3}$

9. Formula for Probable error (PE)

$$PE = 0.6745 \frac{(1-r^2)}{\sqrt{n}}$$

Range of Population r $\Rightarrow r \pm PE$

Q.1 $r = 0.7$, $n = 49$ then $PE \rightarrow 0.049$

D.O.C. $\rightarrow$ 0.7 X = +/- + 1 X 0.6745 ÷ 7 =

10. The method to find two regression equation is called as 'method of least square'.

Example- Find which regression eq. is x on y & y on x?

$$5x - 3y = 12 \qquad\qquad 2x - 3y = 6$$

$$bxy - 4x = a + bxy\,y \qquad\qquad byx - 4y = a + byx\,x$$

1st x on y

$$5x = 3y + 12$$

$$x = \frac{3y}{5} + \frac{12}{5}$$

$$bxy = \frac{3}{5}$$

2nd y on x

$$3y = 2x - 6$$

$$y = \frac{2x}{3} - \frac{6}{3}$$

$$byx = \frac{2}{3}$$

$$bxy \times byx \le 1$$

$$\frac{3}{5} \times \frac{2}{3} \le 1$$

$$0.4 \le 1$$

So, the assumption is right the first equation is x on y and second equation is y on x

Example $4x - 7y = 12$, $x - 5y = 8$

1st y on x 2nd x on y

$Y = a + b_{yx} x$ $X = a + b_{xy} y$

$y = \frac{4x}{7} - \frac{12}{7}$ $x = 5y + 8$

$b_{yx} = \frac{4}{7}$ $b_{xy} = 5$

$$b_{xy} \times b_{yx} \leq 1$$
$$5 \times \frac{4}{7} \leq 1$$
$$2.8 \leq 1$$

Not Satisfied

Assumption 1 is not right so we do the opposite of it

$b_{xy} = \frac{7}{4}$ $b_{yx} = \frac{1}{5}$

$$b_{xy} \times b_{yx} \leq 1$$
$$\frac{7}{4} \times \frac{1}{5} \leq 1$$
$$0.35 \leq 1$$

Assumption 2 is right that equation 1 is x on y and equation 2 is y on x.